

**Mathematics: analysis and approaches****Standard level****Paper 1**

Name

Date: \_\_\_\_\_

1 hour 30 minutes

**Instructions to candidates**

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written in the answer boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

**exam: 9 pages**

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**Section A** (37 marks)

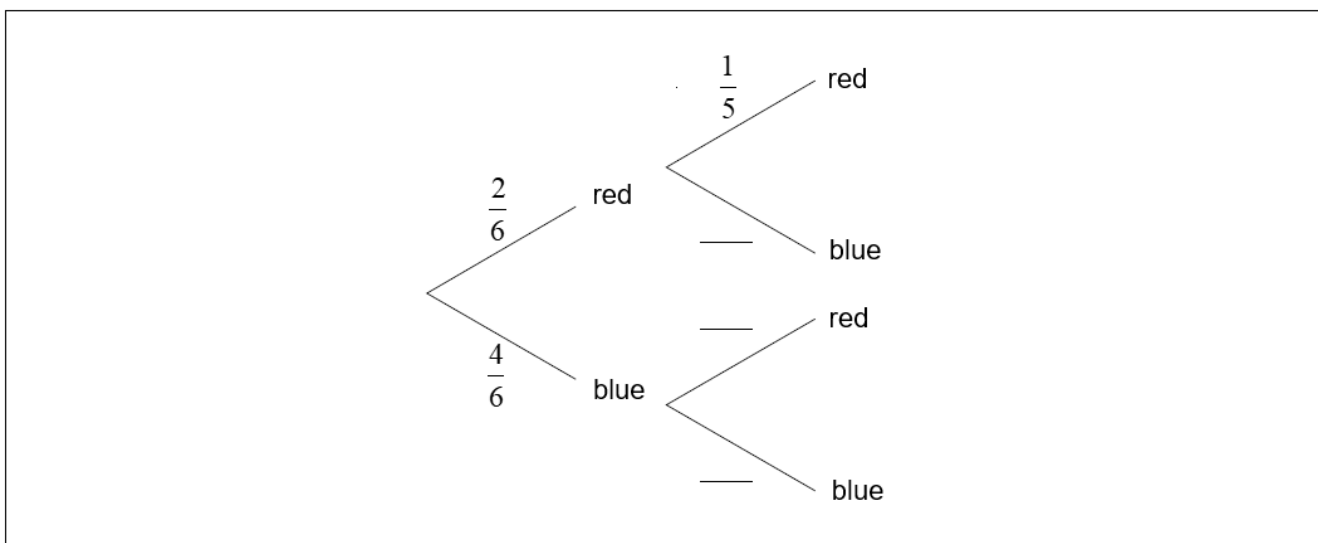
Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

A bag contains 2 red balls and 4 blue balls. Two balls are selected at random without replacement.

(a) Complete the following diagram.

[3]



(b) Find the probability that exactly one of the selected balls is red.

[3]

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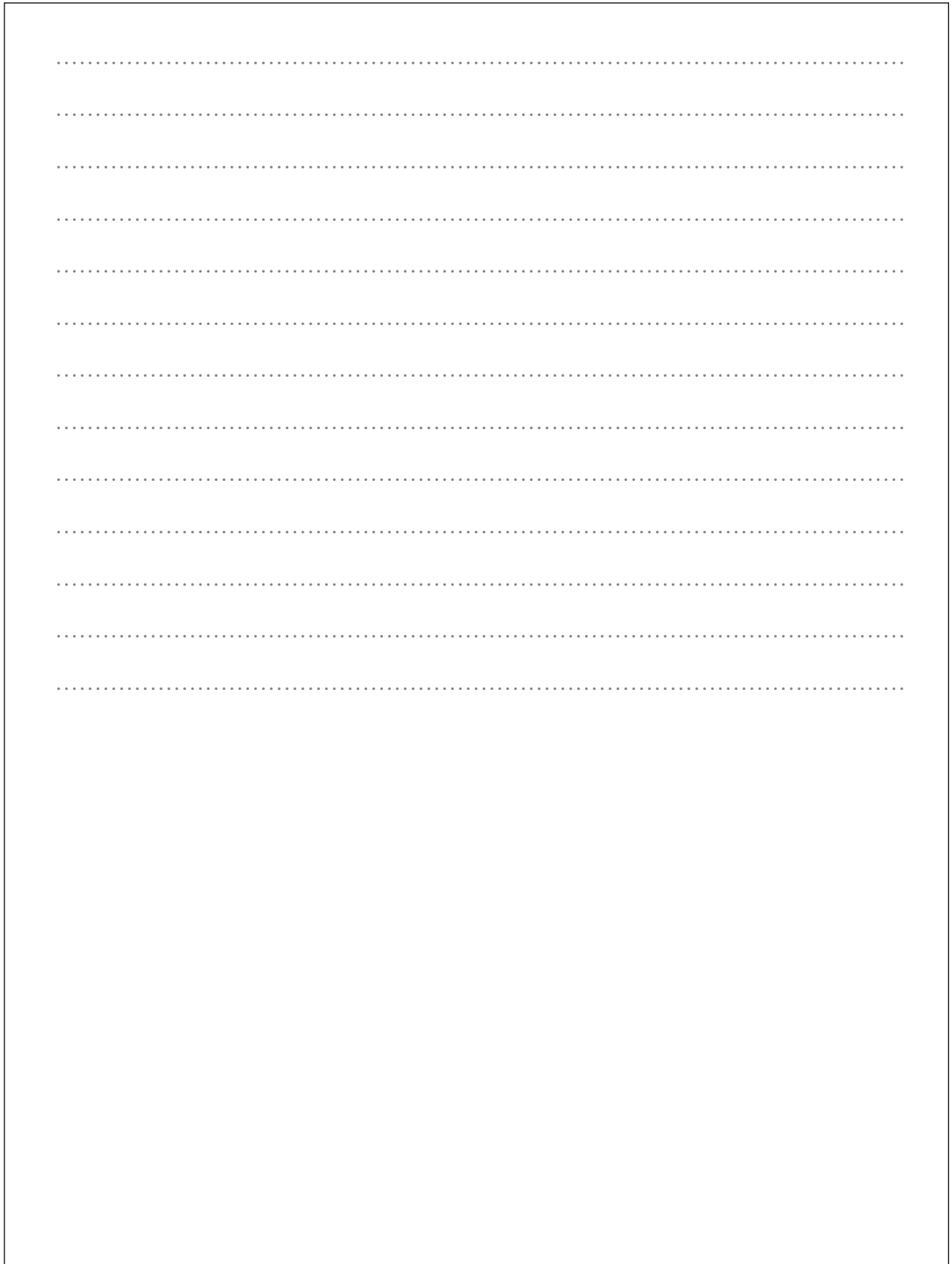
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**2.** [Maximum mark: 4]

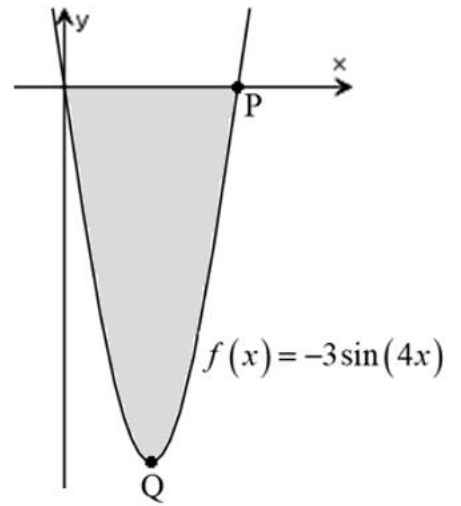
The equation  $ax^2 + 3x + 2 = 0$ , where  $a$  is a constant, has exactly one solution. Find the value of  $a$ .



3. [Maximum mark: 6]

A portion of the graph of  $f(x) = -3\sin(4x)$  is shown.  
The point P is an  $x$ -intercept with coordinates  $(p, 0)$ .

- (a) Find the value of  $p$ . [2]
- (b) The point Q is a minimum. Write down the coordinates of Q. [2]
- (c) Write down a definite integral, but do not evaluate it, that represents the shaded region bounded by  $f$  and the  $x$ -axis. [2]



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**4.** [Maximum mark: 7]

The sum of the first three terms of an arithmetic sequence is 6 and the fourth term is 16.

Find the first term,  $u_1$ , and the common difference,  $d$ , of the sequence.

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5. [Maximum mark: 7]

(a) Given  $f(x) = x^2 + 4x - 10$ ,  $x \leq -2$  show that  $f^{-1}(x) = -2 - \sqrt{x+14}$ ,  $x \geq -14$ . [4]

(b) The graphs of  $f$  and  $f^{-1}$  intersect at point C. Find the coordinates of C. [3]

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## 6. [Maximum mark: 7]

$$\text{Show that } \log_2 \sqrt{8} + \log_b \sqrt{ab} = \frac{\ln(ab^4)}{\ln(b^2)}$$

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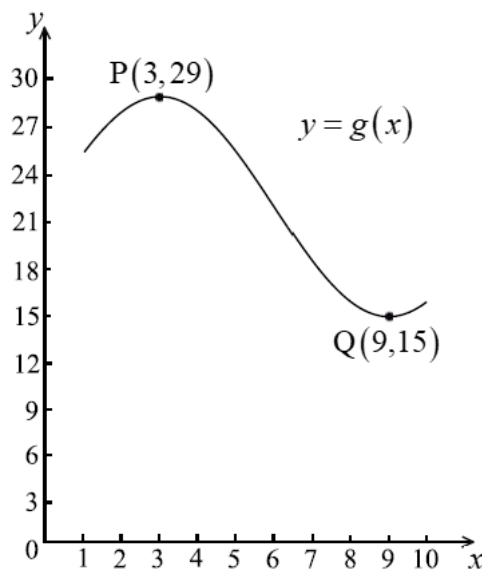
### Section B (43 marks)

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

7. [Maximum mark: 12]

Let  $g(x) = a \cos[b(x+c)] + d$ ,  $1 \leq x \leq 10$ . The graph of  $y = g(x)$  is shown below.

There is a maximum value of 29 at P when  $x = 3$ , and a minimum value of 15 at Q when  $x = 9$ .



(a) (i) Given  $a > 0$ , find the value of  $a$ .

(ii) Show that  $b = \frac{\pi}{6}$ .

(iii) Find the value of  $d$ .

(iv) Write down the value of  $c$ .

[7]

The graph of  $g$  undergoes three transformations. It is stretched horizontally by a scale factor of  $\frac{1}{2}$ , followed by a vertical translation of  $+8$  units, followed by a horizontal translation of  $+6$  units. The new transformed graph is the graph of the function  $h$ .

(b) Find the coordinates for the maximum point on the graph of  $h$ .

[2]

(c)  $h(x)$  can be expressed in the form  $h(x) = g[B(x+C)] + D$ . Find the value of  $B$ , the value of  $C$ , and the value of  $D$ .

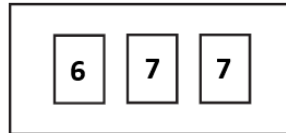
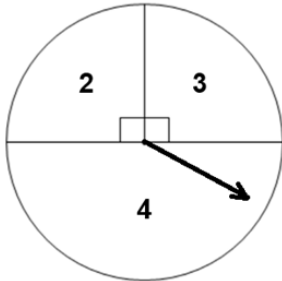
[3]



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8. [Maximum mark: 15]

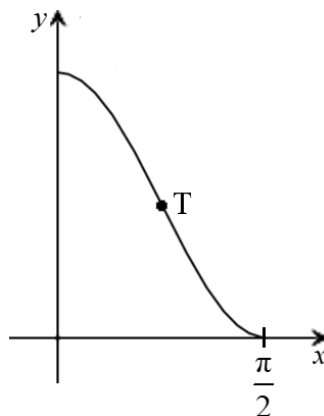
A spinner consists of an arrow that rotates about the centre of a circle so that one of three numbers is randomly selected (see diagram below). There is also a box containing three numbered cards as shown below.  $S$  is the sum of two numbers – one selected randomly with the spinner and the other from randomly selecting one of the cards from the box.



- (a) Write down the four different possible values of  $S$ . [2]
- (b) Find the probability of each value of  $S$ . [5]
- (c) Show that the expected value of  $S$  is  $\frac{119}{12}$ . [2]
- (d) Anna plays a game where she wins \$15 if  $S$  is an even number and loses \$10 if  $S$  is an odd number. Sophie plays the game 12 times. Find the amount of money she expects to have at the end of the 12 games. [6]

9. [Maximum mark: 16]

A graph of the function  $f(x) = 2\cos^2 x$ ,  $0 \leq x \leq \frac{\pi}{2}$  is shown below.



- (a) Point T is a point of inflexion. Show that the coordinates of T are  $\left(\frac{\pi}{4}, 1\right)$ . [5]
- (b) Line L is tangent to the graph of  $f$  at T. Find the equation of L and express it in the form  $y = mx + c$ . [4]
- (c) Find the area of the region bounded by the  $x$ -axis, the  $y$ -axis and the graph of  $f$ . [7]